POLARIS

2016 CONCRETE CANOE
DESIGN REPORT

NAU ASCE

NORTHERN ARIZONA UNIVERSITY

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EXECUTIVE SUMMARY

As the evening sun sets and darkness envelops the city, the residents of Flagstaff can count on an array of bright stars to scatter above the shadowy trees and lighten the night. At seven-thousand feet above sea-level, Flagstaff Arizona's dimmed street lights, low buildings, and clear air provides for a front-row seat to the beautiful night sky and in October 2004, the City of Flagstaff became the world's first International Dark Sky City. Our City's dedication to conserving the night sky inspired this year's concrete canoe theme, *Polaris*, or the Northern Star. Often used for navigation, Polaris guided voyagers across the rough seas to a destination or common goal. As the Northern Arizona University (NAU) Concrete Canoe Team, we collectively strive to present a quality product for all aspects of the Pacific Southwest Conference (PSWC) Concrete Canoe Competition, utilizing collaboration as a guide.

Located in Flagstaff, Arizona, NAU is fortunate to be surrounded by picturesque landscape. In addition to the beautiful dark night skies, NAU lays in a forest of Ponderosa Pines, while sitting below the white snowcapped San Francisco Peaks.

Founded in 1899, NAU has since grown from 23 students to over 25,000, spread amongst seven undergraduate colleges. NAU competes in the competitive PSWC against 18 other schools in which NAU's concrete canoe *Night Fury* took 6th place in 2013, *Spirit* took 13th place in 2014, and *Dreadnoughtus* took 3rd place in 2015.

Based on the success of *Dreadnoughtus* last year, the team decided to continue to use CeraTech's EkkoMAXX™ green cement, proven to be both strong and sustainable. The concrete mix provides an early high compressive strength and is 100% fly ash; therefore it reduces material sent to the landfill while lessening water content and CO_2 use (CeraTech, 2014). Last year's team spent the majority of concrete testing determining best practices and mixing techniques to obtain a consistent trend of data with the new material EkkoMAXX™. Our team was able to work off of this testing, add pigment, and create 20 iterations to find a mix we were confident with. To cure the canoe, the team built a new moisture curing structure in which the canoe was enclosed in a 24' x 8' x 8' wooden structure with four humidifiers. The structure is able to maintain 99% humidity while providing an even distribution of moisture across the canoe.

The mold and hull design from last year's concrete canoe, *Dreadnoughtus*, was reused, and in turn, structural analysis was greatly refined. The team's structural lead programmed various Microsoft Excel sheets to allow a user to change properties such as dimensions, concrete density and loading scenarios to easily calculate properties such as waterline, buoyancy and stresses along the canoe. In addition, NAU has previously analyzed the hull as a rectangular-shaped cross section section, however this year, analysis was refined to a more accurate parabolic shape.

With a total of five members on the team, all new project leads, communication is key to success at the PSWC. Similar to how Polaris guided voyagers, our team guides each other. Without a great deal of previous experience on this project, the team relies on collaboration for further direction and progress, striving to our collective goal.

Table 1: Concrete Canoe Properties

Concrete Canoe Name: Polaris	
Hull Dimensions	
Maximum Length	252 in.
Maximum Width	27.0 in.
Maximum Depth	13.5 in.
Average Thickness	0.5 in.
Weight	175 lbs.
Reinforcement	
Primary	SpiderLath Fiberglass Stainless Steel Post-Tensioning Cable
Secondary	MasterFiber® M 100
Color	
BASF MasterColor: Black (5%)	

Table 2: Concrete Properties

Structural Mix	
Plastic Unit Weight	66.1 pcf
Oven-Dry Unit Weight	59 pcf
28-day Compressive Strength	1950 psi
28-day Tensile Strength	190 psi
28-day Flexural Strength	1230 psi
Concrete Air Content	1.6%
Patch Mix	
Plastic Unit Weight	63.1 pcf
Oven-Dry Unit Weight	58.8 pcf
28-day Compressive Strength	1090 psi
Concrete Air Content	1.0%

PROJECT MANAGEMENT

To achieve success at the Pacific Southwest Conference, the team utilized collaboration as our "Polaris," or our guide; this was achieved by implementing an Integrated Project Delivery (IPD) approach to the construction of our canoe. This approach consists of open and fluid communication amongst all team members to collectively make decisions. By using this approach, leads from different disciplines were able to weigh structural integrity versus constructability versus cost. By understanding the progress of each discipline, the team was able to increase efficiency and reduce wasted time/resources associated with incorrect design or construction work. In addition, the project schedule, estimated budget, risk management plan and safety plan were determined by the project manager prior to design work and construction, then approved by all disciplines.

At NAU, the ASCE Concrete Canoe Competition is offered as a capstone senior design project, therefore team leads are limited to five senior-level students including a project manager, construction manager, structural engineer, concrete mix designer and a reinforcement designer. All of the design and construction of the concrete canoe were completed by the five team leads, a handful of volunteers during canoe casting, and two mentees. The mentee program, in its third year, allows underclassmen students to shadow the current captains and potentially lead future teams. The total person hours for the team leads, mentees and volunteers summed to 1180 hours, distributed amongst project management, hull design, structural analysis, mix design, mold construction, canoe construction, finishing and academics, as shown in Figure 1: Person Hour Breakdown. The largest amount of time was allocated to the canoe construction.

Table 3: Project Milestones

Milestone	Variance	Reason
ASCE NCCC Rule Review	None	None
Concrete Mix Design/ Reinforcement Selection	2 weeks	Further concrete testing
Structural Analysis	2 weeks	Further concrete testing
Canoe Pour	None	None
Canoe Finishing	None	None
Attend ASCE PSWC	None	None

To support the IPD approach, meetings with the project leads and mentees were held twice each week to provide an update of current and upcoming tasks. In addition, there was a portion of the meeting set aside to comment on the progress of each task in reference to the scheduled date of completion, focusing on milestones and the critical path. To determine the critical path, the project manager created a project network diagram in which each node consisted of a task, duration and predecessor. This helped to determine the path with the longest completion time, or the critical path, as seen in Figure 3: Simplified Project Network; the critical path is listed in blue. The critical path was delayed due to the team's decision to perform further concrete testing; although testing was delayed, the pour date milestone was maintained.

The budget for this year's concrete canoe, *Polaris*, relied more heavily on testing and less on construction than previous years. As seen in Figure 2: Budget Allocation and Comparison, last year largely focused on constructing reusable resources, such as the mold and canoe strong-back, and an investment in new paddles and life vests; therefore, by saving money in the mold construction and paddling equipment categories, we were able to allocate more funds to improving the concrete mix design and reinforcement. A much larger array of aggregates were obtained, further discussed in the testing and development section, mix proportions were adjusted and a new reinforcement was implemented.

In order to manage risk, the team members were not only leads of a discipline, but also a secondary lead to a different role. Therefore, if a mistake was made or assistance was needed, a well-informed secondary team member was present for verification and support.

Lastly, as shown in Figure 4: Safety Flow Chart, the team followed careful practices to assure safety through the duration of the testing and construction phases. The following three practices were key in the team's safety plan: a minimum of two people must be present at the concrete lab at all times; protective gear, such as a respirator mask or goggles, must be worn when appropriate; and proper operation of equipment and handling of hazardous materials must be understood and executed.

PROJECT MANAGEMENT RESOURCE ALLOCATION

Figure 1: Person Hour Breakdown Total Hours: 1180

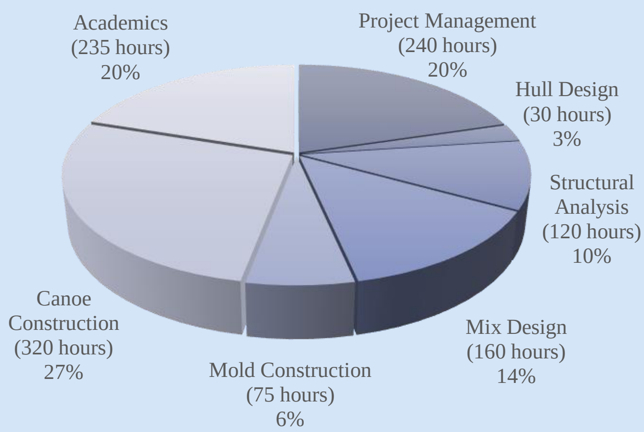


Figure 1: Person Hour Breakdown – The person hour breakdown provides a visual representation of allocation of the team's time. Note that some values are approximated.

Figure 2: Budget Allocation and Comparison Total Cost: $6030

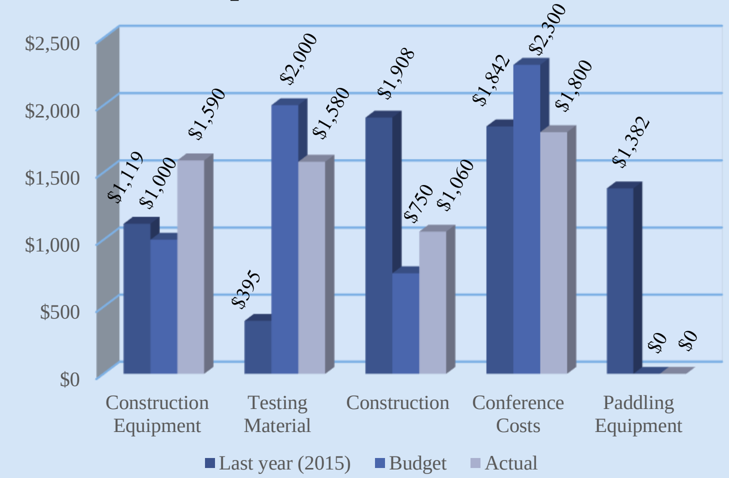


Figure 2: Budget Allocation – The budget allocation provides a comparison of money spent in terms of last year's budget, this year's budget and actual costs. Some values are approximate provided that all items have not yet been purchased.

Figure 3: Project Network

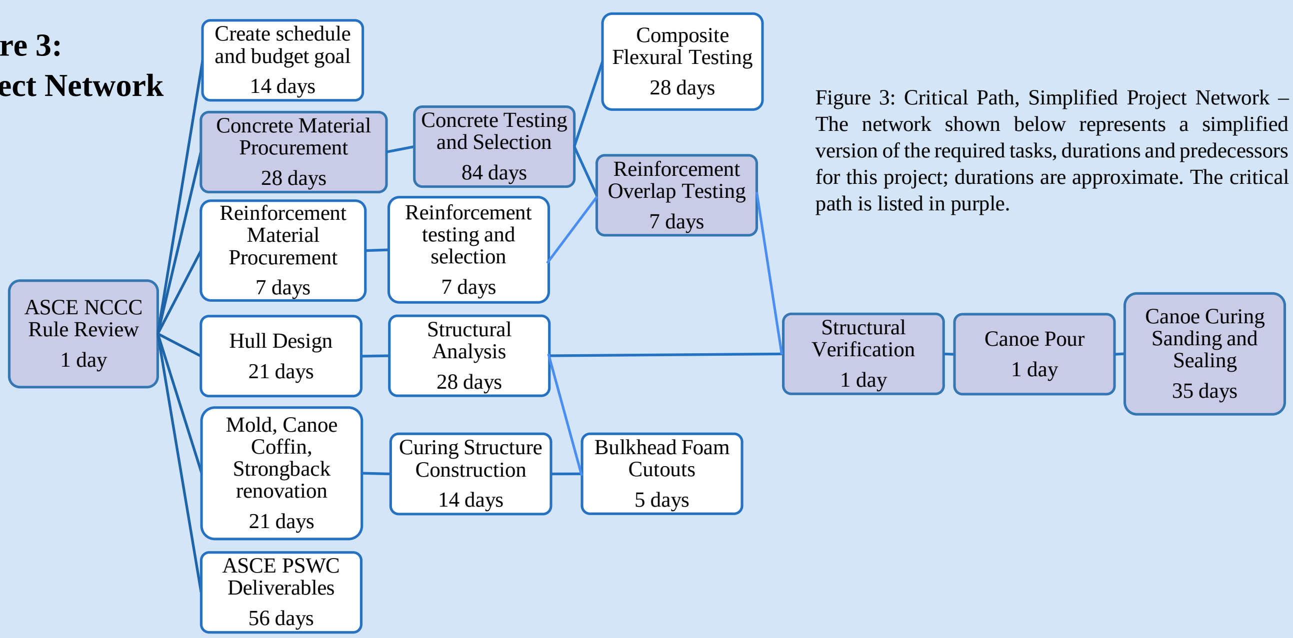


Figure 3: Critical Path, Simplified Project Network – The network shown below represents a simplified version of the required tasks, durations and predecessors for this project; durations are approximate. The critical path is listed in purple.

Figure 4: Safety Flow Chart

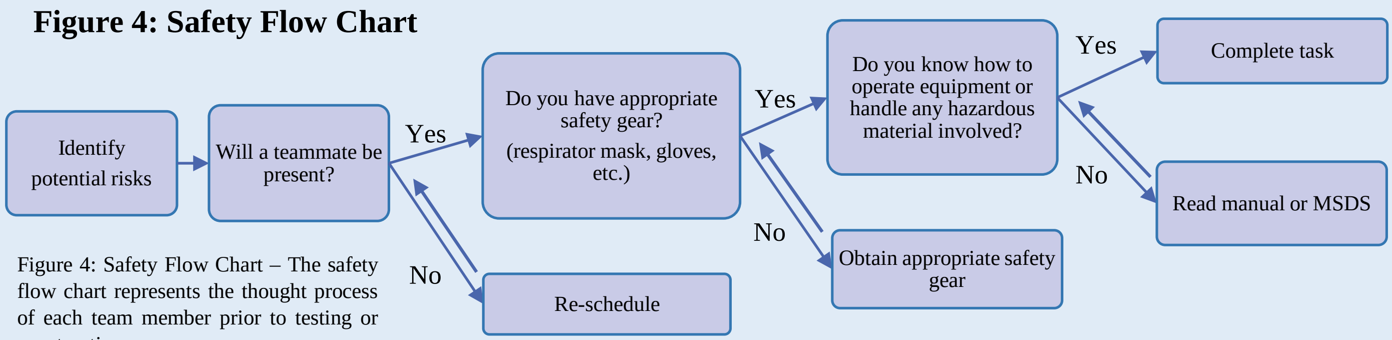


Figure 4: Safety Flow Chart – The safety flow chart represents the thought process of each team member prior to testing or construction.

POLARIS

ORGANIZATIONAL CHART

PROJECT MANAGER

Chelsie Kekaula: Senior
Registered Participant: 2 years

Lead team scheduling, task managmenet, finances and fundraising. Also responsible for material procurement, concrete mix design and testing, graphic design and paddling program. Assisted other tasks as needed.

CONSTRUCTION MANAGER

Colton McConnell: Senior
Registered Participant: 1 year

Managed construction tasks such as mold and strongback renovations, and the construction of a new concrete curing structure.

STRUCTURAL ANALYSIS

Brent Lipar: Senior
Registered Participant: 1 year

Improved structural analysis methods by determining how to perform computer software and hand calculations in a more precise manner.

CONCRETE LEAD

Evan Kaichi: Senior
Registered Participant: 0 years

Researched and tested concrete mix designs, lead material and equipment procurement and assisted construction tasks.

REINFORCEMENT LEAD

Emily Melkesian: Senior
Registered Participant: 2 years

Tested various reinforcement materials, selected a final reinforcement mesh and determined overlap placement.

NAU ASCE

Paddlers

Name	Year	Years as a Registered Participant
Chelsie Kekaula	SR	2
Emily Melkesian	SR	2
Colton McConnell	SR	1
Zach Crimmins	SR	1
Brando Gutierrez	SR	1
Gina Boschetto	SO	1
Ian Connair	SO	1
Paige Reilly	SO	1

Canoe Pour Volunteers

Name	Year
Chris Hazel	SR
Dillon Corrington	SR
Tommy Perkins	JR
Robert Hoppe	JR
Chris Prodan	JR
Gina Boschetto	SO
Kayley Adams	SO
Jimmie McConnell	n/a

Mentees

Name	Year
Stephanie Crocker	JR
Gina Boschetto	SO

HULL DESIGN AND STRUCTURAL ANALYSIS

Last year, NAU's 2015 *Dreadnoughtus* designed their canoe focusing primarily on the optimal hull speed, forgoing additional stability. Due to a mutual agreement amongst all roles part of this year's 2016 Concrete Canoe team, "*Polaris*", it was decided to reuse last year's hull design and focus on fine tuning and automating this year's structural calculations for future NAU teams. This resulted with the construction of a canoe with a maximum hull width, depth, length, and rocker in the bow and stern of 27 in., 13.5 in., 21 ft., 5 in., and 3 in. respectively.

With the use of the Vacanti Yacht Design Software "*Prolines V7*" and the Microsoft Office Software "*Excel 2013*", hydraulic analyses for the waterlines of *Polaris* were performed. The waterlines are designed for the 2-person, 4-person, and fully-submersed load cases. Calculations are designed according to a more accurate cubic function, compared to a linear relationship of the buoyant force versus draft of the canoe, as seen in Figure 5.

The waterline values in Table 4 are based upon the actual weight values for each paddler (reference Appendix C for example calculation) and their specified race. However, when designing for the waterline of last year's conservative 200 pound (lb.) paddlers, the cubic function outputs a lower value when compared to the linear function. This allowed *Polaris* to maximize the canoe's aesthetic appeal, knowing the acquired freeboard is a more precise estimation without causing any excess frictional drag.

Due to *Polaris's* concrete mix having a dry-unit weight of 59 pounds per cubic foot (pcf), bulkheads are not necessary for the canoe to float on water, however, it was determined to create bow and stern bulkheads - 35 in. and 29 in. respectively - to allow 0.2 in. freeboard for the floatation test. This also allowed a factor of safety in case of potential human errors during construction, or the possibility of the canoe not being at its optimum dry-unit weight by the time of conference.

Polaris analyzed the longitudinal and transverse moments along the entire canoe at 1 in. and 6 in. increments respectively, for three different load cases: simply-supported 2-men race, 2-woman race, and 4-person race. All loadings are automated according to various sectional properties obtained through the Autodesk Software "AutoCAD 2015" and obtained weights of the paddlers to

Figure 5: Buoyant Force vs. Waterline

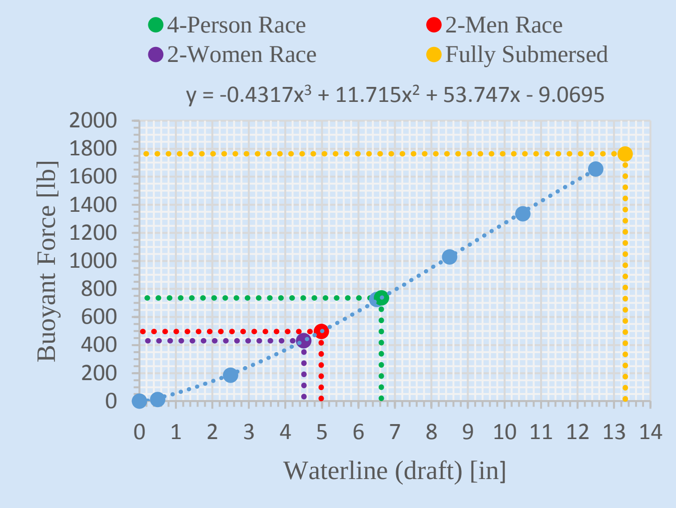


Figure 6: Longitudinal Moment Load Case Comparison

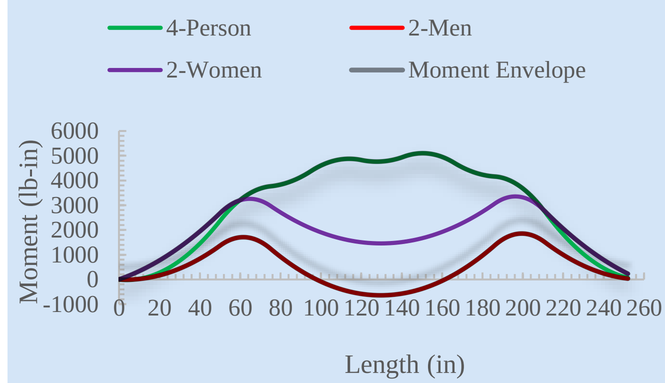


Table 4: Waterline Calculations

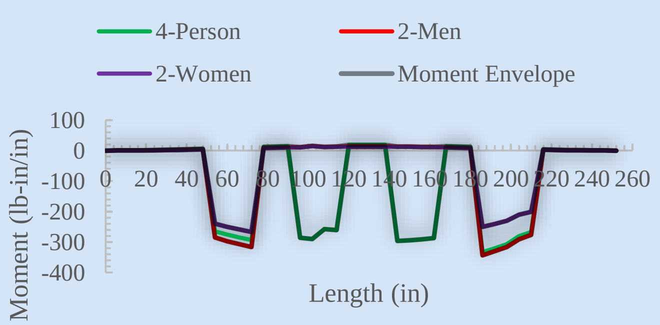
Load Case	Freeboard (in)	Max. Draft (in)
4-Person	6.9	6.6
2-Men	8.5	4.9
2-Women	9.0	4.5
Fully-Submerged	0.2	13.3

calculate the longitudinal moment across the canoe; reference Figure 6 for a comparison of longitudinal moment. The longitudinal loading is based on analyzing the canoe as a simply-supported beam with "supports" at the bow and stern. Through the understanding that the buoyant force is equal but opposite to the weight of the system, the force was applied as an uniformly distributed load across the bottom of the canoe, while the paddlers were applied as two and four distributed loads from the top. Reference Appendix C for an example calculation.

The transverse loading for each cross-section is analyzed as a cantilevered column lengthened to half the exterior curve length of each section. The hydrostatic force being applied to the hull has been designed with a trapezoidal loading according to the draft at each cross-section. This allows a more accurate transverse shear and moment diagram by attaining a load closer to the ideal parabolic load. Figure 7 illustrates the maximum transverse moment for the load cases across the length of the canoe. The drastic change in moment across the length of the canoe is due to the paddler's weight being applied to certain sections. However, this is taken into account by the stiffeners having an effective width of 12 in. to cover the span of the paddlers.

Last year's team, "*Dreadnoughtus*", designed each cross-section as a U-Channel, resulting in a decreased tensile stress in comparison to analyzing each cross-section as a parabolic shape. For *Polaris*, each cross-section was analyzed as a parabolic shape, resulting with a higher maximum tensile stress demand and lower maximum compression stress demand throughout the length of the canoe; as can be seen from Table 5.

Figure 7: Max Transverse Moment Load Case Comparison



Flexural capacities were generated through the use of the Load and Resistance Factor Design (LRFD) method and *ACI 318-14 Standards*. The hull is analyzed as three separate components: 1x1x.5 [in.] panels, WT-shape ribs, and transverse cross-sectional parabolas. After iterating multiple grid reinforcement placements for the panel and rib hull components, it was determined to place the grid 3/8 of an inch into the hull. This was determined to maximize the moment arm of the reinforcement while attaining 1/8 in. of clear cover so the concrete would bond correctly. The transverse cross-sectional areas that experience longitudinal loading are analyzed as parabolas and through the use of the strain-compatibility theory, the flexural and cracking moment capacities were calculated. In Table 5, the demands, capacities, and factor of safeties of the hull components are compared.

To prevent flexural failure and mitigate cracks, six post-tensioning tendons were placed symmetrically about the geometric center of the canoe. The change in post-tensioning losses were taken into account - including curvature frictional losses, wobble losses, anchorage losses, elastic shortening - across the length of the tendon; it was determined that a maximum of 85 pounds (lbs) of tension applied to each strand would be the max tension to apply. This tension force is based off 11 cross-sections including the critical section of the canoe, and the overall constructability of the post-tensioning system. It was partially assumed and calculated that *Polaris* lost approximately 30% of post-tensioning resulting with 57 lbs of tension in each tendon.

Table 5: Comparison of Max Stress Demand, Capacity, and Factor of Safety

Location	Type [1]	Demand (psi)	Capacity (psi)	F.S.[2]
Shear and Flexural				
1" x 1" x 0.5" Panels	T	425.24	1715.9	4.04
	C	425.24	1715.9	4.04
WT-Shape Ribs	T	266.7	5290.6	19.8
	C	266.7	5290.6	19.8
Transverse Cross-Section	T	145.7	917.5	6.3
	C	151.7	1319.5	8.7

[1] Note that Type refers to tension (T) or compression (C)
[2] F.S. means Factor of Safety

DEVELOPMENT AND TESTING

The goal for *Polaris* was to focus on sustainability while building upon the concrete mix design achieved last year. The baseline concrete was selected from last year's canoe, *Dreadnoughtus*, which utilized EkkoMAXX™. EkkoMAXX™ is a "Green cement concrete that offers high early strengths, improved volume stability, and low heat of hydration" (CeraTech 2014). In using EkkoMAXX™, our concrete is 100% fly ash based, due to its poor reactivity with Portland Cement. The lightweight aggregates considered for mix designs were Poraver®: 0.1 – 0.5 mm, 0.5 – 1.0 mm and 1.0 – 2.0 mm and 3M Glass Bubbles: K1, K15, K20, S32, and S35. Prior to performing mix designs, research was completed on all aggregates, cementitious materials, and mix methodologies to optimize this year's lightweight concrete.

The team improved upon the quality control for the concrete mixing process. The procedure incorporated mixing all the cementitious materials, glass bubbles, and fibers in a concrete mixer for 30 seconds. The Poraver® would be hand mixed with half of the batch water and added to the cement mixer for another 30 seconds. The liquid additives would then be added to the mixer slowly and additional water and pigment would be added to achieve a desirable slump. This procedure aided in reducing the amount of clumps that would form if the cement was mixed in a varying procedure.

Many combinations of the considered aggregates were tested. It was decided that the ideal mix design was to use small aggregates to increase the compressive strength while reducing the amount of cementitious material to sustain a lightweight concrete. Designing mixes with smaller aggregate diameters using 0.1 – 0.5 mm of Poraver® provided for smoother concrete but the plastic unit weight was higher than desired. Mix designs with larger Poraver®, 0.5 – 1.0mm and 1.0 – 2.0 mm, provided a courser concrete and the plastic unit weight decreased by at least 5 pcf a cylinder. The team decided to use 0.5 – 1.0 mm Poraver® and combined various glass bubble sizes to continue with the mix designs and find a practical compressive strength. Properties of the aggregates are displayed in Table 6.

Table 6: Concrete Aggregates

Material	S32 Glass Bubbles	K20 Glass Bubbles	Poraver 0.5-1.0mm
Size (mm)	0.08	0.105	0.5-1.0
Specific Gravity	0.32	0.20	0.44
Isostatic Crush Strength (psi)	2000	500	290
Volume in Mix	14.70%	9.00%	36.04%

When narrowing down mix designs, the team found that using lighter Glass Bubbles, K1 or K20, required a larger quantity in the mixes. These lighter Glass Bubbles provided a weaker mix. Using stronger Glass Bubbles, S32, provided stronger concrete, but also slightly increased the plastic unit weight. Figure 8 is a graph showing the compressive strength vs the plastic unit weight for each type of glass bubbles tested.

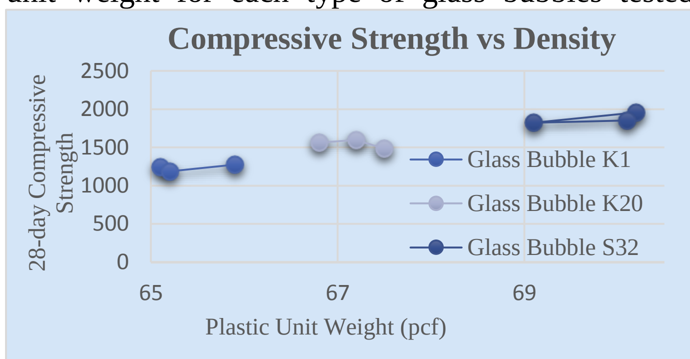


Figure 8: Compressive Strength vs. Density

Figure 9: Concrete Compression Test

Figure 10: Shrinkage Test Molds

Figure 11: Concrete Mixing

After performing 20 mix design and compression tests, as seen in Figure 9, a final mix design was selected using the materials of fly ash, S32 and K20glass bubbles, Poraver® 0.5 – 1.0 mm, and MasterAir AE 90 Air Entrainment. The air entrainment admixture dosage was 3 oz/cwt which provided the best consistency and workability. Compared to the baseline concrete mix design, the volume of fly ash was decreased to 21.2%, while the volume of Glass Bubbles increased to 23.7%. The volume of Poraver® remained unchanged at 36%. The remaining 15% of materials resulted from the two liquid additives and water. The plastic unit weight of the final mix design was 67.4 pcf (ASTM C138) with a slump of 6.5 inches.

Figure 12: Slump Test

Figure 13: Spray Testing

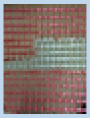
Based on calculations, the air content in the final mix was determined to be 1.6% (ASTM C138). EkkoMAXX™ is known to have reduced shrinkage after 28-days of curing, in comparison to Portland Cement Concrete. The shrinkage of EkkoMAXX™ was tested by placing concrete in a 1-in x 1-in x 10-in rectangular mold, as seen in Figure 10. The specimens were removed 24 hours after the concrete was placed to be cured in a moist environment for 28-days (ASTM C157). Performing shrinkage tests on EkkoMAXX™, concrete with pigment in our final mix shrank 0.04%. The concrete canoe shrank an estimated 0.1 inches after curing for 28-days.

To shotcrete the canoe, hours of spray testing and determining the desired slump was performed for quality control, as displayed in Figures 11,12, and 13. In these tests, different slumps were analyzed and the psi of the air compressor was optimized so that all materials could pass through the nozzle of the sprayer. The fibers in the mix designs were MasterFiber M 100 which measured 0.75 inches in length. The fibers were separated before getting mixed into each batch to assure the fibers were thoroughly distributed throughout the cement. The shorter fibers allowed the sprayers to not clog while the mixture was exiting the nozzle of the sprayer and have an easier time releasing materials while still providing an ideal tensile strength. Two different types of sprayers were tested, Sharpshooter 2.0 and Stucco Mortar sprayer. The concrete could not pass through the Stucco Mortar sprayer, even with varying the psi of the attached air compressor and changing multiple accessory parts of the sprayer. Through testing, it was determined that the ideal sprayer for this year's canoe was the Sharpshooter 2.0. This sprayer was able to spray a consistent layer of concrete, provided the slump was 6 – 10 inches (ASTM C1611). The ideal slump for the canoe was 6-7 inches to be consistent with the analyzed mix designs.

The evaluation of each mix design was based on 4 in. by 8 in. compressive cylinder tests. The tests were performed after curing times of 7, 14, and 28-days. At least two cylinders were broken for each test to obtain an average compressive strength for each mix design. The compressive strength of the final mix design was found to be 1950 psi (ASTM C39) and the tensile strength was 190 psi (ASTM C496).

Although the team desired to continue with the "green" initiative and reuse materials if possible, a stronger material was desired for use as the primary reinforcement within the concrete canoe. A stronger reinforcement was desired to alleviate the potential for cracks within the canoe, as last year's *Dreadnoughtus* had a longitudinal crack running along the bottom of the hull. For *Polaris's* reinforcement, three surplus NAU mesh materials were considered, as well as a new material, SpiderLath Fiberglass Lath System. To determine the optimal reinforcement for the canoe, data was collected for each material's tensile strength and elongation using an Instron 3885 H screw driven machine. From the results, displayed in Table 7, the SpiderLath Reinforcing mesh was selected, due to its high strength, large percent open area (62.6%) for bonding properties, and its workability with the concrete.

Table 7: Reinforcement Comparison

Material	Photo	Strength (lb)	Elongation (in)
SpiderLath Fiberglass Reinforcement Mesh (This year's material)		756	0.25
Parex Glass Fiber Reinforcement Mesh (Last year's material)		72	0.62
TriAx Geogrid (TX140)		135	0.08
Dryvit Reinforcement Mesh		102	0.07

Following ASTM C78/C78M guidelines, a third point loading test was conducted to determine the flexural strength of the composite concrete and SpiderLath reinforcing mesh. The test was completed by applying weights onto the composite samples until failure was reached. The average modulus of rupture for the samples was determined to be equal to 1226.43 psi.

To determine the development length placement of the reinforcement mesh within *Polaris*, three samples of the mesh and concrete were created, each with varying overlap lengths, shown in Figure 14. The lengths selected were 2, 4, and 6 inches. Through testing the samples, it was found that all overlap lengths tested were sufficient for placement in the canoe, as all samples failed within the reinforcement, versus pulling out. For placement in the canoe, the 4 inch overlap was selected, to add an additional factor of safety, although the two inch was sufficient. The reinforcing mesh was applied in 4 feet wide sheets prior to the final 1/8-inch layer of concrete, along with a 4-inch strip along the gunwales, seen in Figure 15. The reinforcing mesh was also placed in 6-inch wide strips along the ribs and center of the canoe after the first 1/8-inch layer of concrete, to minimize the potential for cracking within the concrete.

Figure 14: Overlap Testing

Figure 15: Reinforcement Placement

Both pre-stressing and post-tensioning was considered for implementation within *Polaris*. For ease of constructability, post-tensioning was selected for the concrete canoe, as seen in Figure 16. To implement the system, six post-tensioning strands were created using 1/16'' wire cables encased in 1/8'' nylon tubing, tied together to form a net around the canoe. The net was created so that the strands were placed symmetrically about the geometric center of the canoe, to ensure a moment was not created within the canoe due to the applied tensile forces. To apply the tension within the cables, a turnbuckle and pull-force scale was used, along with a button stopper system. Three button stoppers were placed along the dead-end of the tendons to ensure minimal slippage losses, and two at the live-end due to the confined area for swaging. The calculated 57 pounds of tensile forces was applied to each of the steel tendons after the canoe had moisture cured for 9 days.

Figure 16: Post-tensioning Layout

CONSTRUCTION

Polaris's hull shape is an offspring of the *Dreadnoughtus* (2015) hull shape, which was constructed as a male foam mold. Prior to the mold constructed for *Dreadnoughtus*, a wood-strip female mold was used previously for *Spirit* (2014) and *Night Fury* (2013).This male foam mold, displayed in Figure 17, was made last year to ease form construction for future years. The foam mold was constructed by printing canoe cross sections, transferring these dimensions to plywood, placing the desired length of foam within the cross-section, and cutting out the required with a hot wire, as displayed in Figure 18. The mold is broken into four sections to make transportation and storage easier and more viable. The reason for the creation of this mold is to ease post-tensioning implementation and ease the construction process.

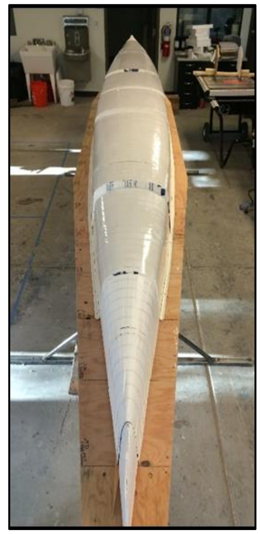


Figure 17: Foam Male Mold

Figure 18: Bulkhead Construction

Prior to pouring *Polaris*, the mold was covered with sheetrock (drywall) joint compound to fill in any imperfections that may show on the inside of the finished canoe. It was then wrapped with industrial shrink-wrap and applied with a heat gun to obtain a finished, smooth surface. The shrink-wrap also allowed the mold to be removed easily after pour day, while keeping it intact, making it so the mold is reusable for future canoes. This mold also has a wooden two by four that runs along the whole bottom side (flat surface) that is indented so it is flush with the foam pieces. The reason for this piece is so that the mold can be easily secured to the canoe table during construction, ensuring that the mold would not shift while the canoe was being poured.

The canoe bulkheads were also constructed in a similar way to the mold itself. The stern bulkhead was calculated to be approximately 2.5 ft. in length, while the bow bulkhead was calculated at 3 ft. in length. The 2 in. thick foam sheets were cut down into smaller square sheets of about one 1 ft. by 1 ft. They were then glued together using spray glue and placed together with weight and gravity. After the sheets were dried, the two wood cross section pieces were clamped together on the outside of the foam and cut in one smooth cut with the hot wire. The very end of the canoe where it comes to a point was then constructed by gluing three sheets together in an opposite direction to the other sheets. Using the hotwire and a combination of skilled eyes and hands the ends were cut by free hand all the way down to a point. Once this was done for both bulkheads sand paper was used to remove any imperfections. Lastly the slots for post tensioning were cut into the foam after the measurements were calculated.

The canoe is post-tensioned with six separate steel cables that are threaded through nylon tubing. These cables were placed on the mold prior to pouring *Polaris* at the correct and calculated distances to ensure the correct placement on pour day. Once all six were placed, thin wire was then wrapped around the nylon tubing across the whole mold in a latitudinal direction to make a six wire net. This net was placed over the second layer of concrete so that the last layers of concrete were poured over top and encased the six wires. These post-tensioning wires were not the only reinforcement used in *Polaris.* Layers of SpiderLath reinforcement were used to withstand forces and help the canoe from buckling. The desired overlap length of 4 in. was taken into account and then the lath was cut into correct sheet size and length for pour day.

The canoe also incorporated a 3-D element into the bow bulkhead and incorporated rib designs for aesthetic purposes, displayed below in Figure 19. The 3-D element, which was placed at the bow bulkhead was created out of plastic with a 3-D printer and displayed a star to represent *Polaris*. The rib designs were formed by using foam letters and shapes as the molds for all four ribs. The ribs spell out Flagstaff, Arizona at Lat (latitude) 35.19° N and Long (longitude) 111.63° W, which is the location of Northern Arizona University and the exact latitude and longitude of where the canoe was constructed.

On pour day, the team arrived and 6:00 am to be absolutely certain that everything was in place for a successful day. Form oil was sprayed over the entire foam mold and especially on the ribs to prevent the concrete from bonding with the mold and to help the demolding process. Concrete mixing was a main concern due to the required slump necessary for the sprayer, and the required timing for placement to ensure cold joints were not present. A half inch slab was poured on the table where the bulkheads were and then the foam bulkheads where placed. The ribs were then filled and packed with concrete prior to the first shotcrete/spray layer. Approximately 1/8 in. of concrete was sprayed over the whole canoe and then reinforcement was placed over the rib sections, between the bulkheads and the rest of the canoe, and one longitudinal strip along the bottom on the canoe. Approximately another 1/8 in. was sprayed again, shown in Figure 20, and rolled into the reinforcement before placing the post-tensioning "net." Another 1/8 in. was then sprayed over the six wires, and then the primary reinforcement was placed over the entire canoe, including the bulkheads. Preceding pour day, all of the reinforcement mesh was cut to its specific sizes, ensuring an ease of placement while constructing the canoe. Concrete was rolled and troweled into the reinforcement to ensure that the concrete would bond correctly and that all imperfections were removed. The last layer of concrete placed on the canoe was also approximately 1/8 in. and was professionally troweled on ensuring a solid and uniform coat.

Figure 19: 3D Element

Figure 20: Concrete Application

After the canoe was poured, an "incubation box" was constructed around it to begin the curing process and a key piece was removed to allow shrinkage of the canoe. The box was constructed out of eleven separate panels, which were created of lumber and tempered hardboard, creating a box that was 24 feet in length, 8 feet wide, and 8 feet tall, as seen in Figure 21. The canoe stayed inside this enclosure with four humidifiers for approximately 4 weeks to moisture cure at 99 percent humidity, until the canoe began air-drying. This incubation system was the first time ever being used at NAU and was a success for curing, constructability, and work area reasons.

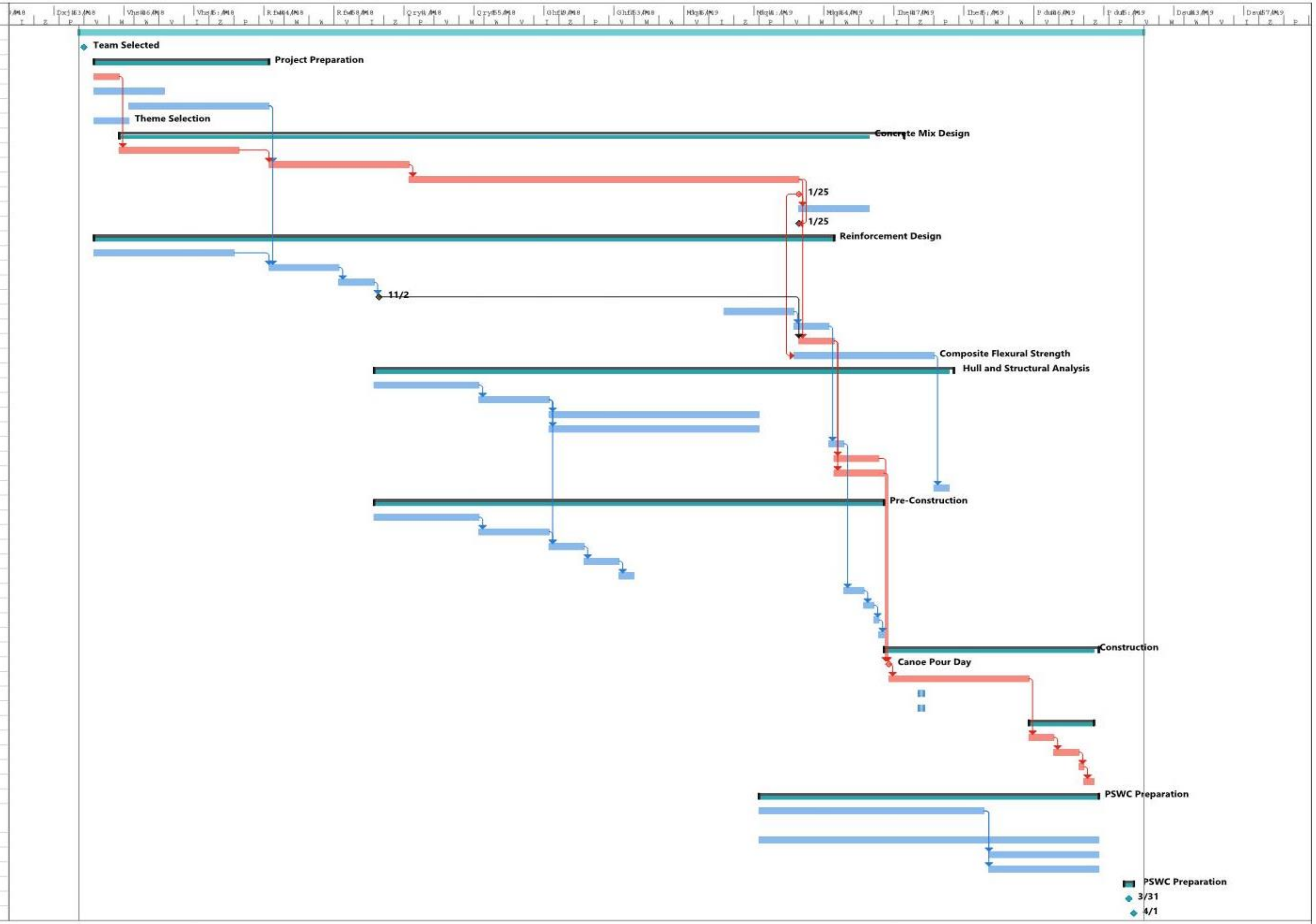
Once initial curing was completed and the mold removed, finishing commenced. Using sanders and diamond polishing equipment, the canoe surface was smoothed. Hydrochloric acid was used to create design in the concrete, and two layers of a cure-sealing compound were used to provide the glossy finish and to reduce water absorption. The lettering for the school name and canoe name were then placed on the outside of the canoe, taking into account placement with the waterline.

Figure 21: Curing Structure Under Construction

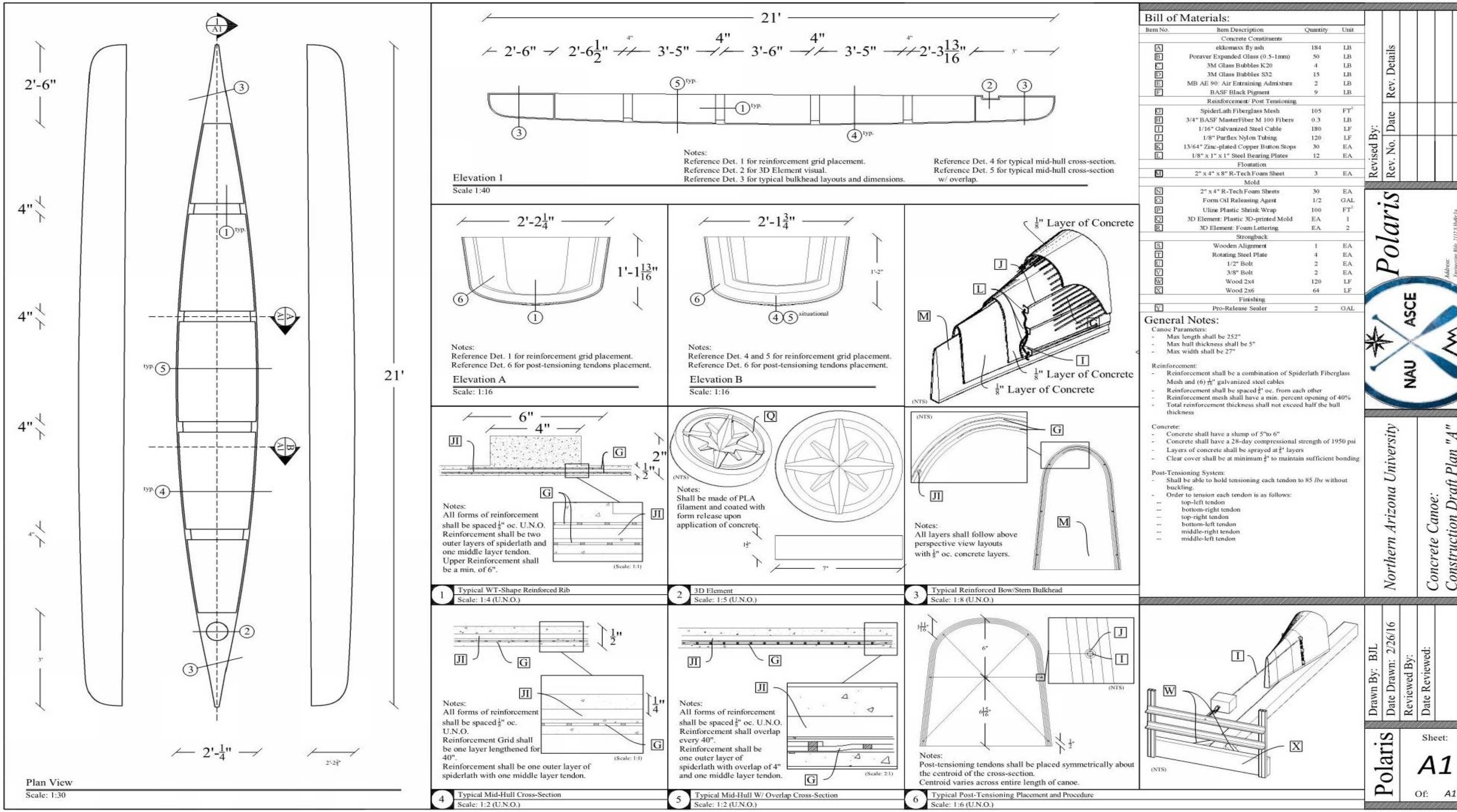
POLARIS

Project Schedule

4	**NAU ASCE 2015-16 Concrete Canoe: Polaris**	**213 days**	**9/4/15**	**4/3/16**
5	**1.0 NAU Concrete Canoe Team Selected**	1 day	9/4/15	9/4/15
6	**2.0 Project Preparation**	**35 days**	**9/7/15**	**10/11/15**
7	2.1 ASCE NCCC Rule Review and Safety Training	5 days	9/7/15	9/11/15
8	2.2 Lab Maintenance and Inventory	14 days	9/7/15	9/20/15
9	2.3 Fundraising	28 days	9/14/15	10/11/15
:	**3.0 Theme Selection**	**7 days**	**9/7/15**	**9/13/15**
;	**4.0 Concrete Mix Design**	**157 days**	**9/12/15**	**2/15/16**
<	4.1 Concrete Material Research	24 days	9/12/15	10/5/15
43	4.2 Concrete Material Procurement	28 days	10/12/15	11/8/15
44	4.3 Concrete Testing	78 days	11/9/15	1/25/16
45	4.4 Concrete Mix Design Selection (from 14-day data	1 day	1/25/16	1/25/16
46	4.5 Concrete Testing	14 days	1/26/16	2/8/16
47	4.6 Obtain 28-day Compressive and Tensile Strength	1 day	1/25/16	1/25/16
48	**5.0 Reinforcement Design**	148 days	9/7/15	2/1/16
49	5.1 Reinforcement Mesh Research	28 days	9/7/15	10/4/15
4:	5.2 Reinforcement Mesh Procurement	14 days	10/12/15	10/25/15
4;	5.3 Reinforcement Testing	7 days	10/26/15	11/1/15
4<	5.4 Reinforcement Selection	1 day	11/2/15	11/2/15
53	5.5 Post-Tensioning System Research and Design	14 days	1/11/16	1/24/16
54	5.6 Post-Tensioning Material Procurement	7 days	1/25/16	1/31/16
55	5.7 Reinforcement Overlap Testing	7 days	1/26/16	2/1/16
56	**6.0 Composite Flexural Strength Testing**	28 days	1/25/16	2/21/16
57	**7.0 Hull and Structural Analysis**	**116 days**	**11/2/15**	**2/25/16**
58	7.1 Canoe Hull Modeling - Prolines	21 days	11/2/15	11/22/15
59	7.2 Bulkhead and Waterline	14 days	11/23/15	12/6/15
5:	7.3 Longitudinal Shear, Bending Moment and Stress	42 days	12/7/15	1/17/16
5;	7.4 Transverse Shear, Bending Moment and Stress	42 days	12/7/15	1/17/16
5<	7.5 Post-Tensioning Analysis, Losses	3 days	2/1/16	2/3/16
63	7.6 Hull Design for Typical Section	9 days	2/2/16	2/10/16
64	7.7 Rib Design and Placement	10 days	2/2/16	2/11/16
65	7.8 Flexural Capacity	3 days	2/22/16	2/24/16
66	**8.0 Pre-Construction**	**102 days**	**11/2/15**	**2/11/16**
67	8.1 Mold Restoration	21 days	11/2/15	11/22/15
68	8.2 Coffin Upgrade	14 days	11/23/15	12/6/15
69	8.3 Foam Bulkhead Construction	7 days	12/7/15	12/13/15
6:	8.4 Curing Structure Construction	7 days	12/14/15	12/20/15
6;	8.5 Test Shotcrete Technique	3 days	12/21/15	12/23/15
6<	8.6 Post-Tensioning System Construction	4 days	2/4/16	2/7/16
73	8.7 Prepare Pour Day Schedule	2 days	2/8/16	2/9/16
74	8.8 Pre-Cut Reinforcement Sheets	1 day	2/10/16	2/10/16
75	8.9 Prepare Dry Batches for Concrete	1 day	2/11/16	2/11/16
76	**9.0 Construction**	**43 days**	**2/12/16**	**3/25/16**
77	9.1 Canoe Pour Day	1 day	2/12/16	2/12/16
78	9.2 Canoe Curing and Daily Humidity Monitoring	28 days	2/13/16	3/11/16
79	9.3 Post-Tension Cables	1 day	2/19/16	2/19/16
7:	9.4 Remove Foam Mold	1 day	2/19/16	2/19/16
7;	9.5 Finishing	13 days	3/12/16	3/24/16
7<	9.5.1 Sand Canoe	5 days	3/12/16	3/16/16
83	9.5.2 Concrete Etch	5 days	3/17/16	3/21/16
84	9.5.3 Apply Lettering	1 day	3/22/16	3/22/16
85	9.5.4 Apply Sealant	2 days	3/23/16	3/24/16
86	**10.0 PSWC Preparation**	**68 days**	**1/18/16**	**3/25/16**
87	10.1 Design Report/ Engineer's Notebook Final Submittal	45 days	1/18/16	3/2/16
88	10.2 Canoe Stand Design and Construction	68 days	1/18/16	3/25/16
89	10.3 Cutaway Section Construction	22 days	3/4/16	3/25/16
8:	10.4 Product/ Table Top Display Construction	22 days	3/4/16	3/25/16
8;	**11.0 ASCE PSWC Conference**	**2 days**	**3/31/16**	**4/1/16**
8<	11.1 Final Product Display and Oral Presentation	1 day	3/31/16	3/31/16
93	11.2 Canoe Races	1 day	4/1/16	4/1/16



Construction Drawing



Bill of Materials:

Item No.	Item Description	Quantity	Unit
	Concrete Constituents		
A	ekkomaxx fly ash	184	LB
B	Poraver Expanded Glass (0.5-1mm)	50	LB
C	3M Glass Bubbles K20	4	LB
D	3M Glass Bubbles S32	15	LB
E	MB AE 90: Air Entraining Admixture	2	LB
F	BASF Black Pigment	9	LB
	Reinforcement/ Post Tensioning		
G	SpiderLath Fiberglass Mesh	105	FT^2
H	3/4" BASF MasterFiber M 100 Fibers	0.3	LB
I	1/16" Galvanized Steel Cable	180	LF
J	1/8" Parflex Nylon Tubing	120	LF
K	13/64" Zinc-plated Copper Button Stops	30	EA
L	1/8" x 1" x 1" Steel Bearing Plates	12	EA
	Floatation		
M	2" x 4" x 8" R-Tech Foam Sheet	3	EA
	Mold		
N	2" x 4" R-Tech Foam Sheets	30	EA
O	Form Oil Releasing Agent	1/2	GAL
P	Uline Plastic Shrink Wrap	100	FT^2
Q	3D Element: Plastic 3D-printed Mold	EA	1
R	3D Element: Foam Lettering	EA	2
	Strongback		
S	Wooden Alignment	1	EA
T	Rotating Steel Plate	4	EA
U	1/2" Bolt	2	EA
V	3/8" Bolt	2	EA
W	Wood 2x4	120	LF
X	Wood 2x6	64	LF
	Finishing		
Y	Pro-Release Sealer	2	GAL

General Notes:

Canoe Parameters:
- Max length shall be 252"
- Max hull thickness shall be 5"
- Max width shall be 27"

Reinforcement:
- Reinforcement shall be a combination of Spiderlath Fiberglass Mesh and (6) 1/16" galvanized steel cables
- Reinforcement shall be spaced 1/8" oc. from each other
- Reinforcement mesh shall have a min. percent opening of 40%
- Total reinforcement thickness shall not exceed half the hull thickness

Concrete:
- Concrete shall have a slump of 5" to 6"
- Concrete shall have a 28-day compressional strength of 1950 psi
- Layers of concrete shall be sprayed at 1/8" layers
- Clear cover shall be at minimum 1/8" to maintain sufficient bonding

Post-Tensioning System:
- Shall be able to hold tensioning each tendon to 85 *lbs* without buckling.
- Order to tension each tendon is as follows:
  - top-left tendon
  - bottom-right tendon
  - top-right tendon
  - bottom-left tendon
  - middle-right tendon
  - middle-left tendon

Appendix A– References

American 3M Center (2013). Technical Data Sheet, 3M™ Glass Bubbles K Series, S Series and iM Series < http://multimedia.3m.com/mws/media/91049O/3m-glass-bubbles-k-s-and-im-series.pdf> (Sep. 15, 2015)

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Northern Arizona University, Concrete Canoe (2013). "*Night Fury*." NCCC Design Paper, Northern Arizona University, Flagstaff, AZ.

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Appendix B–Mixture Proportions Table

Mixture Designation: Structural Mix

Cementitious Material

Component	***Specific Gravity***	***Volume (ft^3)***	***Amount (mass/volume) (lb/yd^3)***	
Cement	n/a	0	c: 0	Mass of all cementitious material cm: 994.0 *lb/yd^3* c/cm ratio: 0
CeraTech EkkoMAXX Flyash	2.78	5.73	m_1: 994.00	

Fibers

Component	***Specific Gravity***	***Volume (ft^3)***	***Amount (mass/ volume) (lb/yd^3)***
BASF MasterFiber M 100 (¾”)	0.91	0.0085	0.50

Aggregates

Aggregates	***Abs (%)***	***MC_{stk} (%)***	***SG***	***Base Quantity (lb/yd^3)***		***Volume SSD (ft^3)***	***Batch Quantity (at MC_{stk}) (lb/yd^3)***
				OD	***SSD***		
Poraver® (0.5-1.0 mm)	20.0	<0.5	0.44	$W_{OD,1}$: 267	$W_{SSD,1}$: 320	9.73	$W_{stk,1}$: 268
3M K20 Glass Bubbles	1.0	0	0.20	$W_{OD,2}$: 30	$W_{SSD,2}$: 30.3	2.43	$W_{stk,2}$: 30
3M S32 Glass Bubbles	1.0	0	0.32	$W_{OD,3}$: 79	$W_{SSD,3}$: 79.8	3.97	$W_{stk,3}$: 79

Admixtures

Admixtures	***lb/gal***	***Dosage (fl.oz/cwt)***	***% Solids***	***Water in Admixture (lb/yd^3)***	
BASF MasterAir AE 90	8.49	3	6.0	1.86	Total Water from All Admixtures 50.90 *$lb//yd^3$*
BASF MasterColor Liquid-Coloring Admixture, Black	15.18	80	48.0	25.84	

Water

	Amount (mass/volume) (lb/yd^3)	Volume (ft^3)
Water (lb/yd^3)	w: 352.0	5.6
Total Free Water From All Aggregates (lb/yd^3)	Σw_{free}: 299.0	
Total Water from All Admixtures, (lb/yd^3)	Σw_{admx}: 25.9	
Batch Water, lb/yd^3	w_{batch}: 349.9	

Densities, Air Content, Ratios and Slump

	cm	fibers	aggregates	solids	water	Total
Mass of Concrete, M (lb, for 1 yd^3)	994.0	0.5	376.0	27.2	349.9	M: 1747.6
Absolute Volume of Concrete, *V*, (ft^3)	5.73	0.01	16.13	0.90	4.23	V: 27

Theoretical Density, T, (=M/V)	64.13 *lb/yd^3*	Air Content [(T – D)/D x 100%)]	1.6 %
Measured Density, D	66.51 *lb/yd^3*	Slump, Slump flow	6.5 in
water/cement ratio, w/c:	0	Water/cementitious material ratio, w/cm	0.36

POLARIS

Mixture Designation: Patch Mix				
Cementitious Material				
Component	***Specific Gravity***	***Volume (ft^3)***	***Amount (mass/volume) (lb/yd^3)***	
Cement	n/a	0	c: 0	Mass of all cementitious material cm: 1040.83 *lb/yd^3* c/cm ratio: 0
CeraTech EkkoMAXX Flyash	2.78	6.00	m_1: 1040.83	

Fibers			
Component	***Specific Gravity***	***Volume (ft^3)***	***Amount (mass/ volume) (lb/yd^3)***
BASF MasterFiber M 100 (¾”)	0.91	0.0085	0.50

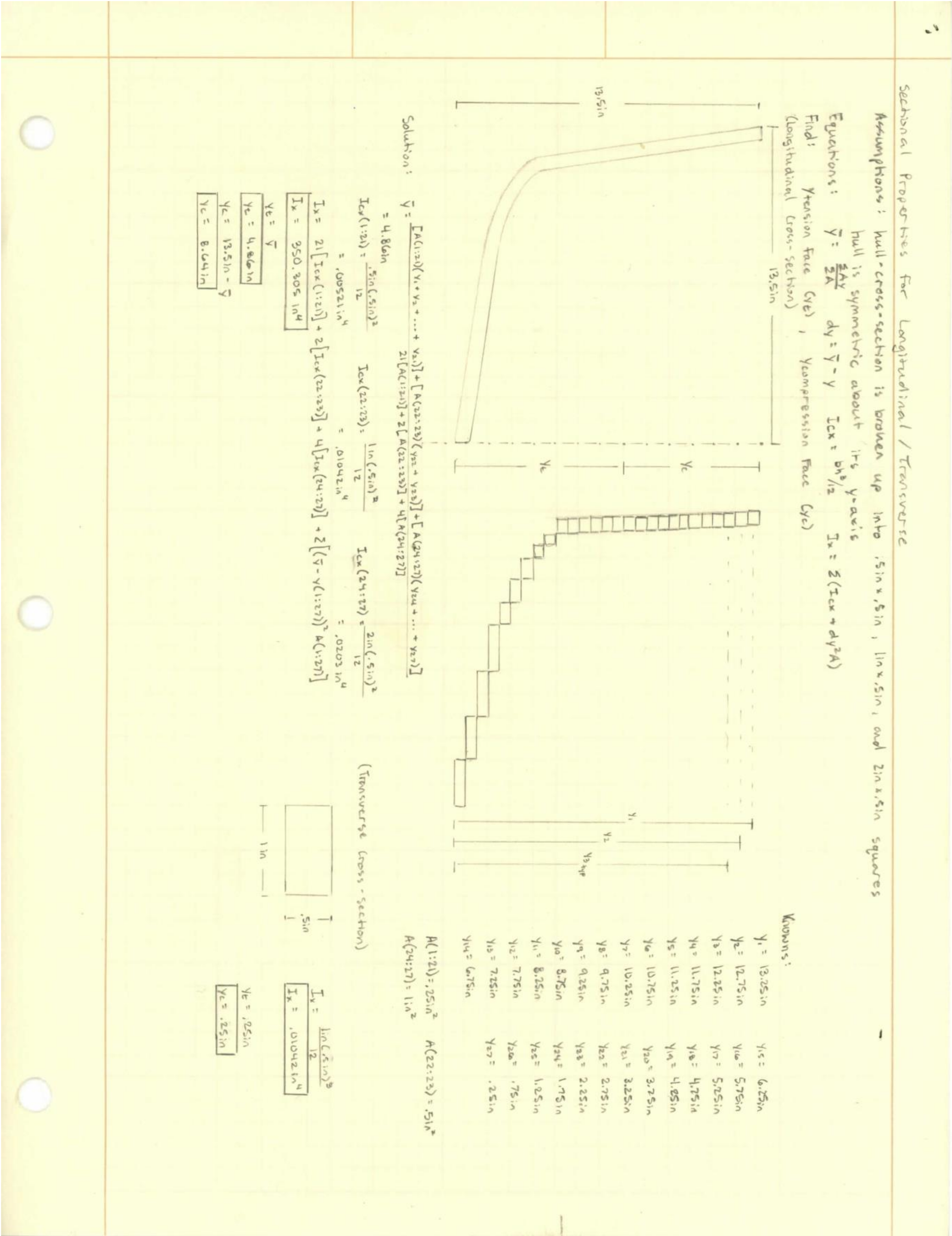
Aggregates							
Aggregates	***Abs (%)***	***MC_{stk} (%)***	***SG***	***Base Quantity (lb/yd^3)***		***Volume SSD (ft^3)***	***Batch Quantity (at MC_{stk}) (lb/yd^3)***
				OD	***SSD***		
3M S32 Glass Bubbles	1.0	0	0.32	$W_{OD,3}$: 312.9	$W_{SSD,3}$: 316.0	15.7	$W_{stk,3}$: 312.9

Admixtures					
Admixtures	***lb/gal***	***Dosage (fl.oz/cwt)***	***% Solids***	***Water in Admixture (lb/yd^3)***	
BASF MasterColor Liquid-Coloring Admixture, Black	15.18	112	48.0	37.9	Total Water from All Admixtures 37.9 *$lb//yd^3$*

Water		
	Amount (mass/volume) (lb/yd^3)	Volume (ft^3)
Water (lb/yd^3)	w: 296.7	5.1
Total Free Water From All Aggregates (lb/yd^3)	Σw_{free}: 293.6	
Total Water from All Admixtures, (lb/yd^3)	Σw_{admx}: 37.9	
Batch Water, lb/yd^3	w_{batch}: 331.5	

Densities, Air Content, Ratios and Slump						
	cm	fibers	aggregates	solids	water	Total
Mass of Concrete, M (lb, for 1 yd^3)	1040.8	0.5	312.9	34.9	331.5	M: 1720.6
Absolute Volume of Concrete, *V*, (ft^3)	6.0	0.01	15.67	0.91	4.41	V: 27
Theoretical Density, T, (=M/V)	63.7 *lb/yd^3*		Air Content [(T – D)/D x 100%)]			0.95 %
Measured Density, D	63.1 *lb/yd^3*		Slump, Slump flow			5 in
water/cement ratio, w/c:	0		Water/cementitious material ratio, w/cm			0.32

Appendix C–Example Structural Calculations



Sectional Properties for Longitudinal / Transverse

Assumptions: hull-cross-section is broken up into .5in x .5in, 1in x .5in, and 2in x .5in squares
hull is symmetric about its y-axis

Equations: $\bar{y} = \frac{\Sigma A y}{\Sigma A}$ $d_y = \bar{y} - y$ $I_{cx} = bh^3/12$ $I_x = \Sigma(I_{cx} + d_y^2 A)$

Find: $y_{tension\ face}$ (y_t), $y_{compression\ face}$ (y_c)

(Longitudinal cross-section)

Knowns:

y_1 = 13.25in	y_{15} = 6.25in
y_2 = 12.75in	y_{16} = 5.75in
y_3 = 12.25in	y_{17} = 5.25in
y_4 = 11.75in	y_{18} = 4.75in
y_5 = 11.25in	y_{19} = 4.25in
y_6 = 10.75in	y_{20} = 3.75in
y_7 = 10.25in	y_{21} = 3.25in
y_8 = 9.75in	y_{22} = 2.75in
y_9 = 9.25in	y_{23} = 2.25in
y_{10} = 8.75in	y_{24} = 1.75in
y_{11} = 8.25in	y_{25} = 1.25in
y_{12} = 7.75in	y_{26} = .75in
y_{13} = 7.25in	y_{27} = .25in
y_{14} = 6.75in	

$A(1{:}21) = .25in^2$ $A(22{:}23) = .5in^2$
$A(24{:}27) = 1in^2$

Solution: $\bar{y} = \frac{[A(1{:}21)(y_1 + y_2 + \ldots + y_{21})] + [A(22{:}23)(y_{22} + y_{23})] + [A(24{:}27)(y_{24} + \ldots + y_{27})]}{21[A(1{:}21)] + 2[A(22{:}23)] + 4[A(24{:}27)]}$

$= 4.86in$

$I_{cx}(1{:}21) = \frac{.5in(.5in)^2}{12} = .00521in^4$ $I_{cx}(22{:}23) = \frac{1in(.5in)^2}{12} = .01042in^4$ $I_{cx}(24{:}27) = \frac{2in(.5in)^2}{12} = .0203in^4$

$I_x = 21[I_{cx}(1{:}21)] + 2[I_{cx}(22{:}23)] + 4[I_{cx}(24{:}27)] + 2[(\bar{y} - y(1{:}27))^2 A(1{:}27)]$

$I_x = 350.305\ in^4$

$y_t = \bar{y}$

$y_t = 4.86in$

$y_c = 13.5in - \bar{y}$

$y_c = 8.64in$

(Transverse cross-section)

$I_x = \frac{1in(.5in)^3}{12}$

$I_x = .01042in^4$

$y_t = .25in$

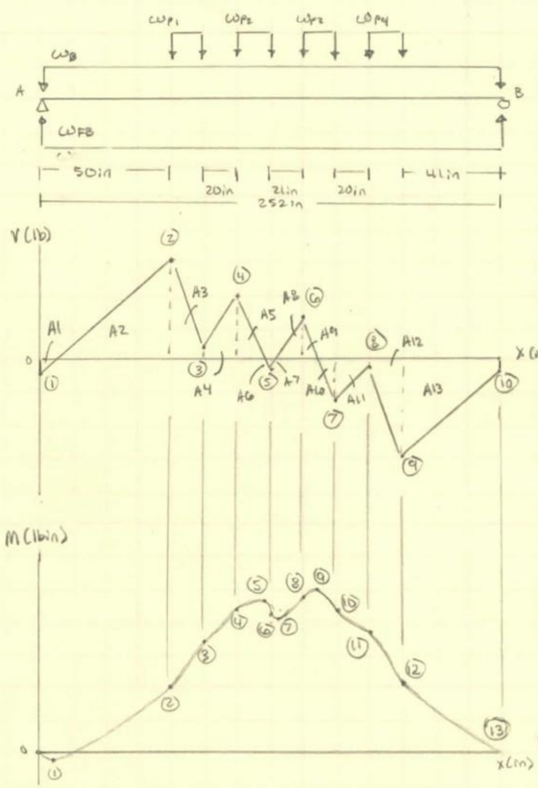
$y_c = .25in$

Longitudinal Load Case

Assumptions: Paddlers and Buoyant Force are Uniform Distributed Loads

Designed as a simply-supported beam with a "pin-, roller-support"

Equations: $F_B = W_{sys}$ $\quad W_{sys} = P_1 + P_2 + P_3 + P_4 + W_c$ $\quad W_c = \gamma_c Vol_c$ $\quad$ Equilibrium Eqns: $\rightarrow \Sigma F_x = 0$ $\quad +\uparrow \Sigma F_y = 0$ $\quad \circlearrowleft \Sigma M = 0$



Knowns: $P_1 = 150\text{lb}$ $P_2 = 120\text{lb}$ $P_3 = 135\text{lb}$ $P_4 = 160\text{lb}$ $\quad \gamma_c = 59\text{ pcF}$

Knee-to-toe span = 25 in $\quad Vol_c = 2.87\text{ Ft}^3$

Solution:

$W_c = 59\text{pcF}(2.87\text{Ft}^3) = 169.33\text{lb}$ $\quad \omega_c = \frac{169.33\text{lb}}{252\text{in}} = .6719\text{ lb/in}$ $\quad W_{sys} = 150\text{lb} + 120\text{lb} + 135\text{lb} + 160\text{lb} + 169.33\text{lb} = 734.33\text{lb}$ $\quad \omega_{FB} = \frac{734.33\text{lb}}{252\text{in}} = 2.914\text{ lb/in}$

$\omega_{P1} = \frac{150\text{lb}}{25\text{in}} = 6\text{ lb/in}$ $\quad \omega_{P2} = \frac{120\text{lb}}{25\text{in}} = 4.8\text{ lb/in}$ $\quad \omega_{P3} = \frac{135\text{lb}}{25\text{in}} = 5.4\text{ lb/in}$ $\quad \omega_{P4} = \frac{160\text{lb}}{25\text{in}} = 6.4\text{ lb/in}$

$\circlearrowleft M_A: 0 = W_{sys}\left(\frac{252\text{in}}{2}\right) - P_1(62.5\text{in}) - P_2(107.5\text{in}) - P_3(153.5\text{in}) - P_4(198.5\text{in}) + R_B(252\text{in}) - W_c\left(\frac{252\text{in}}{2}\right)$

$= 13.0952\text{ lb}$

$\circlearrowleft M_B: 0 = P_1(189.5\text{in}) + P_2(144.5\text{in}) + P_3(98.5\text{in}) + P_4(53.5\text{in}) + W_c\left(\frac{252\text{in}}{2}\right) - W_{sys}\left(\frac{252\text{in}}{2}\right) - R_A(252\text{in})$

Shear:

0-50: $y = 2.914\text{ lb/in}\,x - .6719\text{ lb/in}\,x - 15.2183\text{lb}$
$y(50\text{in}) = 96.8867\text{lb}$ ①

50-75: $y = 2.914\text{ lb/in}\,x - .6719\text{ lb/in}\,x - 6\text{ lb/in}\,x + 96.8867\text{lb}$
$y(25\text{in}) = 2.9392\text{ lb}$ ②

75-95: similar process
$y(20\text{in}) = 47.7812\text{lb}$ ③

95-120: similar process
$y(25\text{in}) = -16.1663\text{ lb}$ ④

120-141: similar process
$y(21\text{in}) = 30.9178\text{ lb}$ ⑤

141-166: similar process
$y(25\text{in}) = -48.0297\text{lb}$ ⑥

166-186: similar process
$y(20\text{in}) = -3.1877\text{ lb}$ ⑦

186-211: similar process
$y(25\text{in}) = -107.1352\text{ lb}$ ⑧

211+252: similar process
$y(41\text{in}) = -15.2091\text{ lb}$ ⑨

Moment:

$A1 = \left[\frac{1}{2}(15.21831\text{b})\right]0.787\text{in} = -51.643\text{ lbin}$ (1)

$A(1{:}2) = \left[\frac{1}{2}(96.886\text{lb})\right]13.679\text{in} + A1 = 2041.724\text{ lbin}$ (2)

$A(1{:}3) = \left[\frac{1}{2}(93.947\text{lb}) + 2.921\text{lb}\right]25\text{in} + A(1{:}2) = 3289.547\text{ lbin}$ ③

$A(1{:}4)$ = similar process $= 3796.751\text{ lbin}$ ④

$A(1{:}5)$ = similar process $= 4243.004\text{ lbin}$ ⑤

$A(1{:}6)$ = similar process $= 4191.91\text{ lbin}$ ⑥

$A(1{:}7) = 4133.63\text{ lbin}$ ⑦

$A(1{:}8) = 4346.808\text{ lbin}$ ⑧

$A(1{:}9) = 4498.166\text{ lbin}$ ⑨

$A(1{:}10) = 4263.037\text{ lbin}$ ⑩

$A(1{:}11) = 3750.863\text{ lbin}$ ⑪

$A(1{:}12) = 2371.327\text{ lbin}$ ⑫

$A(1{:}13) = 0\text{ lbin}$ ⑬

Shear @ mid-hull = 2,714 lb

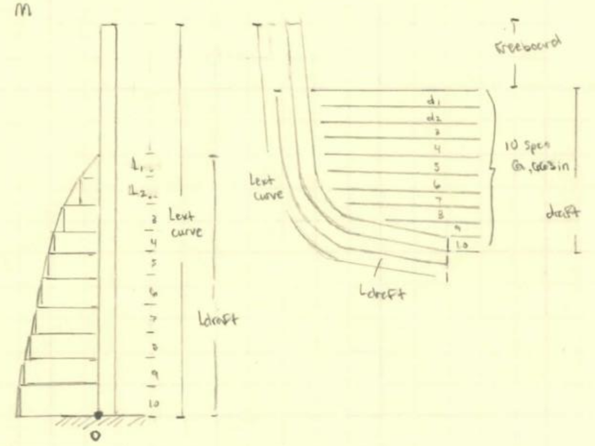
Moment @ mid-hull = 4118.98 lb

Transverse Load Case

Assumptions: designed as a fixed cantilever

Equations: $\omega_B = \gamma_w (\text{draft})(\text{unit length})$ Equilibrium Eqns: $\rightarrow \Sigma F_x = 0$ $\uparrow \Sigma F_y = 0$ $\circlearrowleft \Sigma M = 0$

Find: M



Knowns: $L_1 = .6671$ in, $L_2 = .6676$ in, $L_3 = .6685$ in, $L_4 = .6699$ in, $L_5 = .6726$ in, $L_6 = .7924$ in, $L_7 = 1.065$ in, $L_8 = 1.5996$ in, $L_9 = 2.4355$ in, $L_{10} = 6.5506$ in

$d_1 = .663$ in, $d_2 = 1.326$ in, $d_3 = 1.989$ in, $d_4 = 2.652$ in, $d_5 = 3.316$ in, $d_6 = 3.979$ in, $d_7 = 4.642$ in, $d_8 = 5.305$ in, $d_9 = 5.968$ in, $d_{10} = 6.627$ in, $\gamma_w = .034$ pcin

Solution: $W_{b1} = \gamma_w d_1 (1) = .0226$ lb/in, $W_{b2} = \gamma_w d_2 (1) = .04528$ lb/in, $\omega_{b3} = \gamma_w d_3 (1) = .0679$ lb/in, $\omega_{b4} = \gamma_w d_4 (1) = .3906$ lb/in

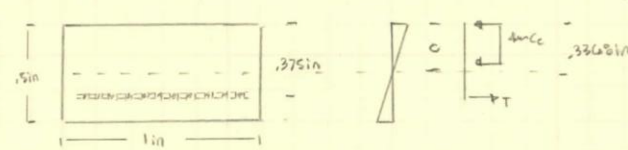
$W_{b5} = .1132$ lb/in, $W_{b6} = .1358$ lb/in, $\omega_{b7} = .1585$ lb/in, $\omega_{b8} = .1811$ lb/in

$W_{b9} = .2037$ lb/in, $W_{b10} = .2263$ lb/in

$\circlearrowleft \Sigma M_o$: $M_o = \Sigma$ [all spaces in this Format $[\frac{1}{2}(W_b)(L_i)(L_{draft} - \frac{2}{3}L_i - \text{prev }L)] - [W_b(L)(L_{draft} - \frac{1}{2}L - \text{prev }L)]$]

$M_o = 12.666$ lbin/in

Flexural Capacity:



$B_1 = .85$ $f'_c = 1950$ psi

$f_y = 85.24$

$A_{grid} = .005568$ in²

$c = \frac{A_s F_y}{.85 f'_c B_1 b_w}$

$c = .3368$ in (neutral axis)

$M_n = T(d - \frac{B_1 c}{2})$

$= 110.03$ lbin

$\phi M_n = .65(110.03$ lbin$)$

$= 71.518$ lb/in > 12.666 lbin

good

Internal Stresses:

Equations: $J = \frac{V}{A_g}$ $\sigma_T = \frac{M y_c}{I}$ $\sigma_C = \frac{M y_c}{I}$ (longitudinal) (transverse)

Knowns: $A_g = 23.11$ in²

V @ mid-hull = 2.714 lb

M @ mid-hull = 4118.985 lbin

} Longitudinal values

Solution:

Longitudinal Stress:

$J = \frac{2.714 \text{ lb}}{23.11 \text{ in}^2}$ **$J = .1175$ psi**

$\sigma_T = \frac{4118.985 \text{ lbin}(4.86\text{in})}{350.305 \text{ in}^4}$ **$\sigma_T = 57.145$ psi**

$\sigma_C = \frac{4118.985 \text{ lbin}(8.64\text{in})}{350.305 \text{ in}^4}$ **$\sigma_C = 101.59$ psi**

Transverse Stress:

$\sigma_T = \frac{12.666 \text{ lbin}(.25\text{in})}{.01042 \text{ in}^4}$ **$\sigma_T = 425.24$ psi**

$\sigma_C = \frac{12.666 \text{ lbin}(.25\text{in})}{.01042 \text{ in}^4}$ **$\sigma_C = 425.24$ psi**

** Reference Longitudinal Calcs / Sectional Property Calcs **